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Terrain Generation Using Fractal Methods

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Terrain Generation Using Fractal Methods

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Abstract

Euclidean geometry is not practical for describing complex natural shapes. A real-time solution using a fractal geometry approach is presented. First, existing digital terrain elevation data (DTED) is analyzed, and then it is interpolated to generate a terrain surface of arbitrarily high spatial resolution. Although fractal geometry provides only an estimate, statistical properties are preserved.

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1. Introduction

Fractal geometry is a mathematical language providing for concise descriptions of natural shapes. In particular, Mark [1] has shown that many geomorphic surfaces can be modeled by a stochastic fractal based on fractional Brownian motion (fBm); fBm is a generalization of classical Brownian motion, where differences between successive positions are normally distributed. However, it is necessary to restrict a single fBm function to some range of scale when simulating a topographic surface.

This report describes an application of this theory to terrain modeling. The approach follows from the work of Yokoya et al. [2]. The next three sections examine fBm approximation, where mathematical purity is sacrificed for realism and execution speed. Section 5 provides a specific example.

Algorithms for the previous and first level digital terrain elevation data (DTED-0) binary data extraction [3] were written in C and C++, respectively. This native code was then integrated with a graphical user interface (GUI) and a three-dimensional (3D) scene graph interactive display using Java technology. In particular, the GUI was done using Java with Swing components, and the 3D computer graphics were written in Java3D. The Java Native Interface (JNI) allows for native code to invoke the Java Virtual Machine (JVM) [4]. Java was chosen because of its "write once, run anywhere" feature with an acceptable response time.

2. Fractional Brownian Function

A stochastic function $f(\mathbf{x})$ constructs fractional Brownian motion if, for all $\mathbf{x}$ and $\mathbf{\Delta x}$,

$$\Pr\left(\frac{f(\mathbf{x}+\mathbf{\Delta x})-f(\mathbf{x})}{||\mathbf{\Delta x}||^H} < t\right) = F(t), \tag{1}$$

where H is a constant that lies in the range $0 < H < 1$ and $F(t)$ is a cumulative distribution function of a random variable t. Bold-faced type indicates a vector quantity; e.g., $\mathbf{x}$ is the position vector to point x and $||\mathbf{\Delta x}||$ is the length of vector $\mathbf{\Delta x}$. Since we are only concerned with 3D surfaces, the fractal dimension of the distribution is determined by

$$D = 3 - H. \tag{2}$$

For our particular application, equation (1) is the probability that the difference in elevation over some scaled distance is less than t.

By assuming that $F(t)$ is a zero-mean Gaussian distribution of variance σ^2, Yokoya et al. [2] rewrites equation (1) as

$$\log E[|f(\mathbf{x}+\mathbf{\Delta x}) - f(\mathbf{x})|] = H \log ||\mathbf{\Delta x}|| + \log C, \tag{3}$$

where $E[|f(\mathbf{x}+\mathbf{\Delta x}) - f(\mathbf{x})|]$ is the expected value, or simply the mean, of the difference of function values over some distance $||\mathbf{\Delta x}||$, and $C = 2\sigma/\sqrt{2\pi}$ (see [2], p. 286). Note that for constant H and C, a log-log plot of $E[|f(\mathbf{x}+\mathbf{\Delta x})-f(\mathbf{x})|]$ as a function of $\|\mathbf{\Delta x}\|$ has the slope-intercept form of a straight line. This is called a fractal plot.

3. Fractal Analysis

We cannot expect a geomorphic surface to have the characteristics of a true stochastic fractal. An exact fBm process is self-affine, a property that is inconsistent with most natural surfaces. In contrast to a self-similiar function where scale factors of independent variables are the same, self-affine functions have different scale factors. A natural fractal, such as terrain, should be approximated by one or more fBm functions, each having a range of scale.

Yokoya describes a technique for determining scale limits $[\|\mathbf{\Delta x}\|_{min}, \|\mathbf{\Delta x}\|_{max}]$ between which a natural fractal can be well-described by a single fBm function. The lower limit $\|\mathbf{\Delta x}\|_{min}$ simply corresponds to the spatial resolution of the original data, which is assumed to be uniform and rectangular. The upper limit $\|\mathbf{\Delta x}\|_{max}$ is determined by a linearity test of the fractal plot. For a two-dimensional (2D) fractal plot, the measure of linearity is given by the following expression:

$$I = \frac{\sqrt{4\mu_{11}{}^2+(\mu_{20}-\mu_{02})^2}}{\mu_{20}+\mu_{02}}, \tag{4}$$

where μ_{20} is the variance of $\log\|\mathbf{\Delta x}\|$, μ_{02} is the variance of $\log E[|f(\mathbf{x}+\mathbf{\Delta x})-f(\mathbf{x})|]$, and μ_{11} is the covariance of the set of n points. The upper limit of scale is computed as

$$\|\mathbf{\Delta x}\|_{max} = \|\mathbf{\Delta x}\|_{min} + n^* - 1, \tag{5}$$

where n^* is the minimum of n that provides the local peak of linearity (see [2], pp. 288-291).

Least-squares regression analysis is then used to estimate H, which is the slope of the limited fractal plot. The fractal dimension follows from equation (2). Finally, the standard deviation $\sigma = C\sqrt{2\pi}/2$ can be calculated since we know $\log C$, the intercept of the least-square line. The parameter H and the standard deviation σ of $F(t)$ for some range of scale $[\|\mathbf{\Delta x}\|_{min}, \|\mathbf{\Delta x}\|_{max}]$ are referred to as the fractal-based features.

4. Fractal Interpolation

Assuming that statistical properties of a natural fractal are preserved at smaller scales, Yokoya et al.[2] extends a range of scale to $[0, \|\mathbf{\Delta x}\|_{max}]$. His stochastic interpolation algorithm, called recursive midpoint displacement, effectively adds a weight to the average of four neighboring values in computing the midpoint. A weight is the product of extracted fractal features and a random element.

Fractal features H and σ are determined by fractal analysis of the original data (see section 3). It is assumed that an N x M rectangular grid of input data is aligned and uniform. The values $f(i,j)$, where both i and j are odd natural numbers satisfying $1 \leq i < 2N$ and

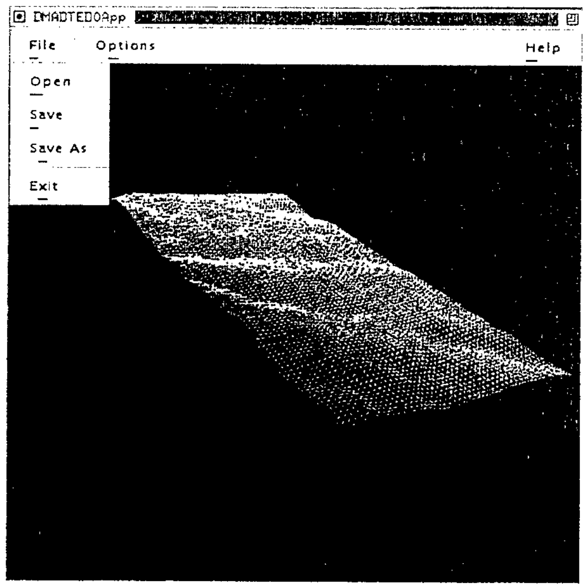

Figure 1: Polygonal surface.

$1 \leq j < 2M$, represent the known surface. First, compute values where both i and j are even:

$$f(i,j) = \frac{1}{4}[f(i-1)(j+1) + f(i-1)(j-1) + f(i+1)(j+1) + f(i+1)(j-1)] + [\sqrt{1-2^{2H-2}} * \|\mathbf{\Delta x}\|^H * \sigma * Gauss()]. \tag{6}$$

When only i or j is even, the values are calculated as

$$f(i,j) = \frac{1}{4}[f(i-1)(j) + f(i)(j+1) + f(i)(j-1) + f(i+1)(j)] + [2^{-H/2}\sqrt{1-2^{2H-2}} * \|\mathbf{\Delta x}\|^H * \sigma * Gauss()]. \tag{7}$$

The function Gauss() returns a Gaussian random number with zero mean and unit variance, i.e., $N(0,1)$. The refined grid at this level of recursion is $(2N-1)$ x $(2M-1)$. Iteration of this procedure results in data of the desired resolution.

5. An Application

A wireframe display for a 10 x 5 km region of eastern USSR is shown in Figure 1; the precise location is $54^0 0'0"$ N latitude, $36^0 0'0"$ E longitude. The 100-m resolution grid of DTED is aligned and uniform. The terrain surface is approximated by 10,000 planar triangles which were determined by tessellation of the DTED. * Intermediate heights are bilinearly interpolated from neighboring elevation posts. Affine transformations of the surface provide the desired view.

*Optimal triangulation was used to avoid generating triangles with long edges and sharp angles.

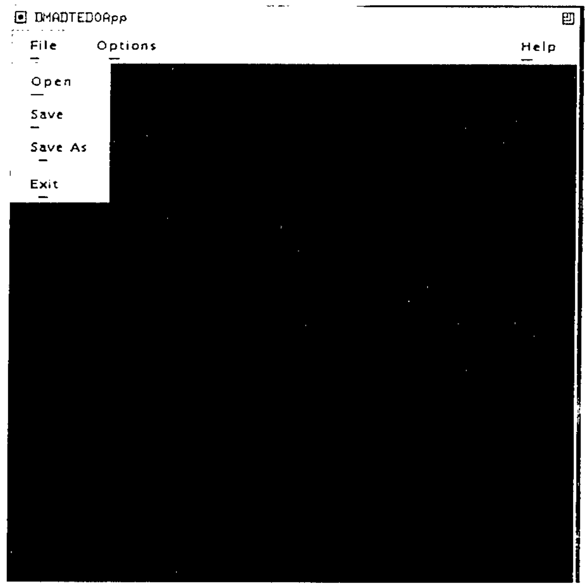

Figure 2: Approximated fBm surface.

The program provides for either a wireframe display or Gouraud solid area scan conversion. Figure 2 shows a shaded display of the same region. In this case, the original terrain area was sampled every 200 meters instead of every 100 meters. The refined surface was then generated from one level of recursive midpoint displacement.

Although the human eye/brain combination is capable of reducing large quantities of data quickly, it is not as capable in discerning detail. A simple way to compare the actual terrain against the fractally generated terrain is a difference statistic. This statistic measures the closeness of two datasets that are in perfect register (i.e., uniform and aligned with elevation values attached to the same spatial coordinates). For the previous example, the original DTED has $\mu_o = 233.088$ and $\sigma_o = 14.735$, and the computed fractal terrain has $\mu_f = 233.060$ and $\sigma_f = 14.568$. This results in a difference statistic with parameters $\mu_d = 0.913$ and $\sigma_d = 1.098$, which suggests high correlation.

Recall that the parameters H and σ characterize a fBm function ranging over $[\|\mathbf{\Delta x}\|_{min}$, $\|\mathbf{\Delta x}\|_{max}]$ (see section **3**). This same function is assumed to hold for the range $[0, \|\mathbf{\Delta x}\|_{min}]$ and allows for processing data at higher levels of resolution if desired.

6. Significance

Digital terrain databases are needed for analyzing many different battlefield activities in a simulation environment, e.g. planning routes of attack and determining line of sight (LOS). Many simulators use low resolution data due to financial and/or hardware constraints. This limitation may be addressed through the proposed terrain synthesis of Yokoya et al.[2].

Our software includes this procedure for real-time fractal interpolation once two parameters are computed for a specific area - the fractal dimension and standard deviation. This approach also allows for real-time alteration of terrain within this dynamic environment, e.g. artillery bombing which may effect the soldier's decision making.

7. Future Work

More rigorous statistical testing should be considered when comparing actual data against simulated terrain data. One possibility is a Kolmogorov-Smirnov (KS) test; KS is a test of distributions and thus more flexible than a simple difference statistic. Also being considered is a Student's t-test, where individual parameters such as the mean and variance are more closely examined.

The computer program was built on a Silicon Graphics O2 visual workstation running IRIX 6.5 version of the UNIX operating system. Supporting system software includes a GNU C/C++ compiler/translator (v2.8.1), which is necessary for compiling native code, and Java2 Execution Environment (Software Development Kit v1.2.1) with a Java3D API (v1.1.1). We are currently porting this software to a microcomputer having a Win32 GNU C/C++ compiler/translator (available free of charge from Cygnus, Inc. at www.cygwin.com) and Java2 1.3, Java3D 1.2 APIs. An effort is now being made to generate relocatable dynamic link libraries (DLLs) analogous to shared objects (.so) in a UNIX environment.

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